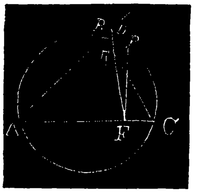
184. Proposed by ERWIN MARTIN, Principal of Schools, Mead, Neb.

If from any point in the circumference of a circle circumscribed about a triangle, perpendiculars are drawn to the sides, or the sides produced, of the inscribed triangle, the lines connecting the feet of the perpendiculars are collinear.

Demonstration by J. R. NITT, Coronal Institute, San Marcos, Tex.; G. I. HOPKINS, A.M., High School, Manchester, N. H.; and G. W. GREENWOOD, B. A., McKendree College, Lebanon, Ill.



Let ABC be a triangle; PD, PE, PF perpendiculars from any point P of circumcircle upon the sides.

To prove D, E, F collinear: Draw BP and CP, DE and DF. The quadrilaterals $BEPD$, $CFPD$ are cyclic.

$\therefore\ \angle PDE=\angle PBE$, $\angle PDF=\angle PCF$. But $\angle PBE=\angle PCF$, each being measured by $\frac{1}{2}$arcAP.

$\therefore\ \angle PDE=\angle PDF$. $\therefore\ D$, E, F are collinear.

Also demonstrated by *BEULAH FRAZIER*, Soph., Rolla School of Mines, and *G. B. M. ZERR*.

162. Proposed by J. D. PALMER, Providence, Ky.

Given the distances from the vertices of a triangle, ABC, to the center of the incircle, to construct the triangle.

Solution by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

Already two solutions of this problem have appeared in the MONTHLY, an erroneous one in the January number, page 15, and a revised solution in the February number, pages 45-46, yet a word or two more may not be superfluous.

The problem is to *construct* a triangle knowing the distances from the incenter to the vertices.

The correct analysis of the problem, as Zerr shows in his revised solution results in a cubic equation, which means that the problem is insoluble, *i. e.* insoluble in the same sense that the trisection of an angle is impossible. The true answer is that the construction called for can not be made by elementry geometry.

The following is a trigonometrical analysis of the problem.

If A, B, C are the angles of a plane triangle, then

$$2\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C+\sin^2\tfrac{1}{2}A+\sin^2\tfrac{1}{2}B+\sin^2\tfrac{1}{2}C=1\ldots.(1).$$

This theorem readily results from substituting in the following well known relation

$$\cos A+\cos B+\cos C-1=4\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C$$

for $\cos A$, $1-2\sin^2\frac{1}{2}A$, etc., and reducing.

Now let α, β, γ be the distances from the incenter to the vertices and r the radius of the inscribed circle; whence

$$\sin\tfrac{1}{2}A=r/\alpha,\quad \sin\tfrac{1}{2}B=r/\beta,\quad \sin\tfrac{1}{2}C=r/\gamma.$$

Substituting in (1), we have